FIRM:DesignCalcs, Inc. JOB NO.2012083 SHEET NO: 1
MADE BY:KJH DATE:03-25-2013 CHECKED BY: DATE:
TITLE:Example Torsion Calculation

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TORSIONAL PROPERTIES:
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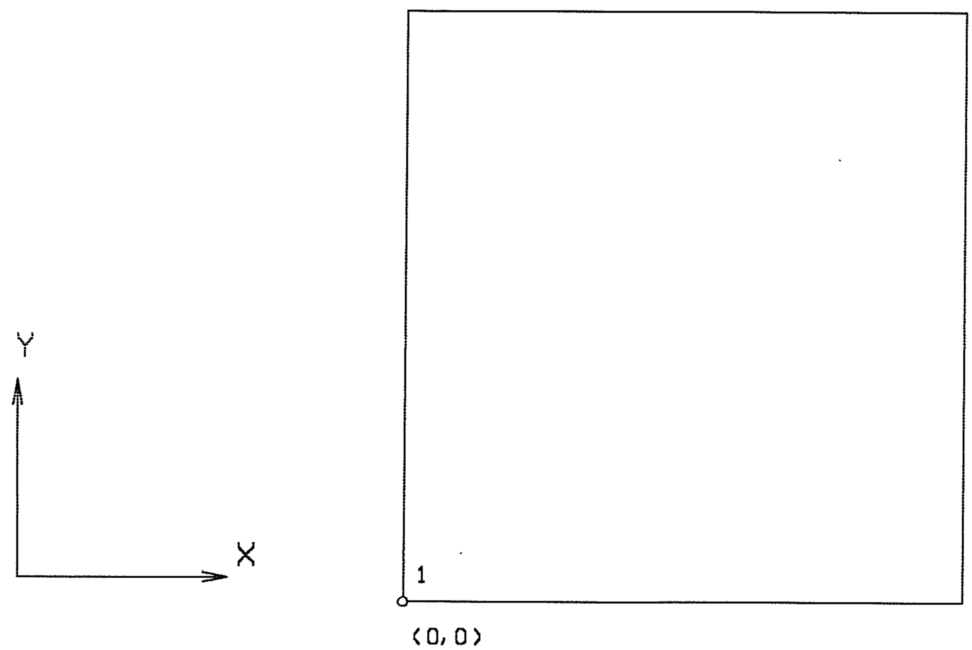


Theory:
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Design Reference:"Theory of Elasiticity" by Timoshenko and Goodier

This program determines the torsional properties for any solid using an iterative finite difference method. The result is the torsional stiffness property, "J", to be used in the following equation,

$\upsilon$ = T/(J*G)

where

$\upsilon$ = angle of twist per unit length
T = Torsion
G = Modulus of elasticity in shear

The torsional properties were first determined by St. Venant in 1855. In 1903, Prandtl proposed a reformulation of the torsional problem which is based on a function,"", called the stress function with the following characteristics,

$\tau_{xz} = \delta \varphi / \delta y$ and $\tau_{yz} = -\delta \varphi / \delta x$ (Chapter 10, Eq. 149)

Using the stress function, the torsional problem becomes a Poisson differential equation of the form,

$\delta^2 \varphi / \delta x^2 + \delta^2 \varphi / \delta y^2 = -2*G*\upsilon$ (Chapter 10, Eq. 150)

Boundary Conditions:
Along all exterior boundaries we set = 0.

Along all interior void boundaries we set the slope, /dn, so that
Integral around void, ( $d\varphi$ /dn )ds = -2*G*$\upsilon$*(Void Area) (App. Eq. 30)

After solving the stress function, the torsional constant, J, is computed,
J = M

## TORSIONAL PROPERTIES:

### Description of the Iterative Solution Method:

1. The solid is discretized into uniformly spaced nodes.
2. Stress function, $\varphi=0$, are assigned at exterior boundaries (this is called a Dirchlet boundary condition).
3. At interior void boundaries, the slope $\partial\varphi/dn$ is known (this is called a Neumann boundary condition.)
4. A computation molecule is applied to all nodes inside the boundaries:

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                    (φi,j+1)
                       |
(φi-1,j)------(φi,j)---------  (φi+1,j)
                       |
                    (φi,j-1)
```

Computational Molecule

For nodes spaced equidistantly,

$$\varphi_{i,j} = 1/4 * [\varphi_{i-1,j} + \varphi_{i+1,j} + \varphi_{i,j-1} + \varphi_{i,j+1}] + DX^2/4*(2*G)$$

5. After solving for all stress functions, shears are computed:

$$\tau_{xz} = (\varphi_{i,j+1} - \varphi_{i,j-1})/(2*DY)$$
$$\tau_{yz} = -(\varphi_{i+1,j} - \varphi_{i-1,j})/(2*DX)$$

6. The torsional constant is computed as,

$$J = 2*\sum\varphi_{i,j}*DX*DY$$

TORSIONAL PROPERTIES:

SHAPE DEFINITION TABLE:

| Shape No. | Description | X (ft) | Y (ft) | R (ft) |
|---|---|---|---|---|
| 1 | Solid Polygon | 0.000 | 0.000 | |
| | | 0.000 | 10.000 | |
| | | 10.000 | 10.000 | |
| | | 10.000 | 0.000 | |
| | | 0.000 | 0.000 | |

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TORSIONAL PROPERTIES:
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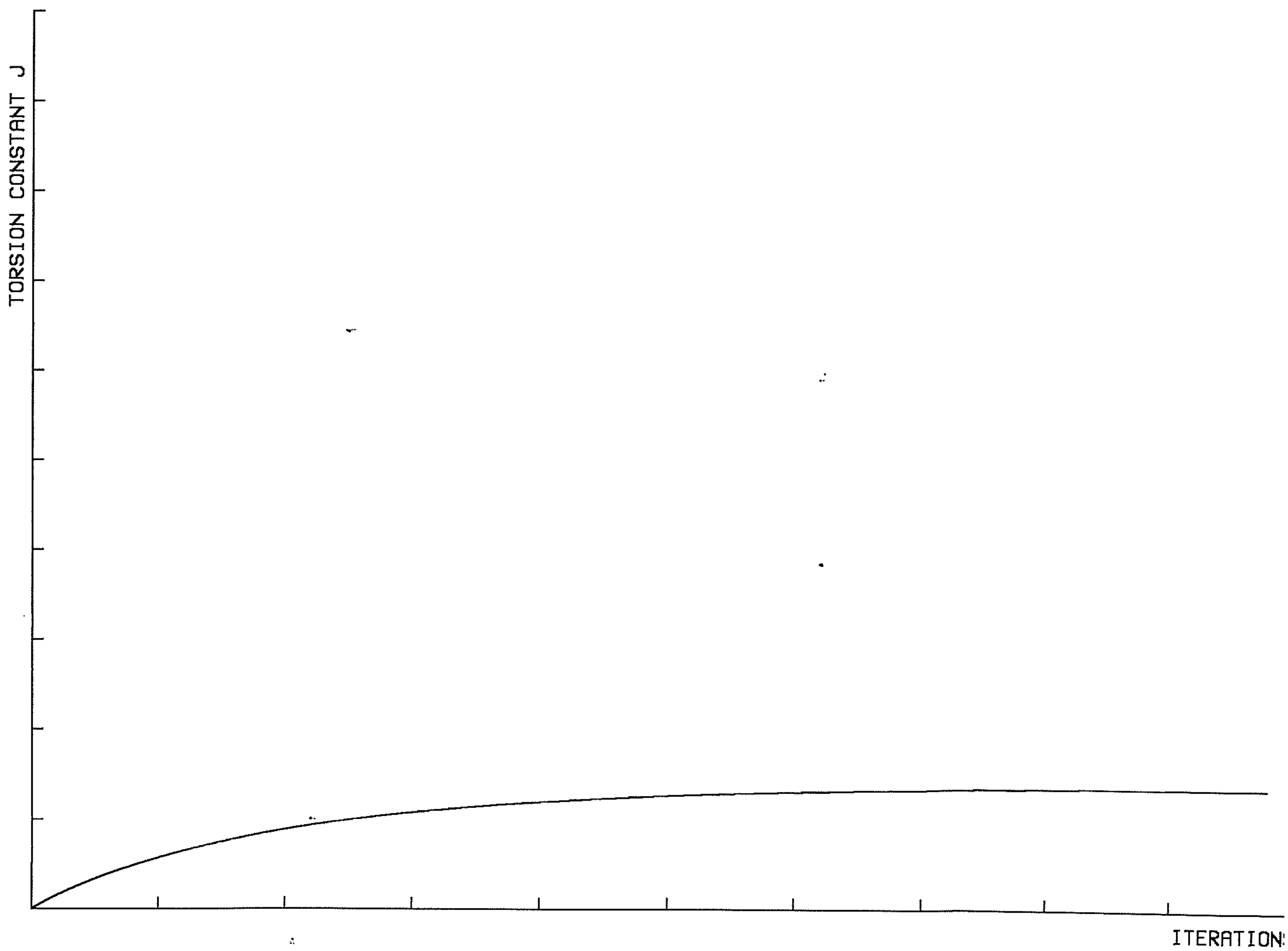
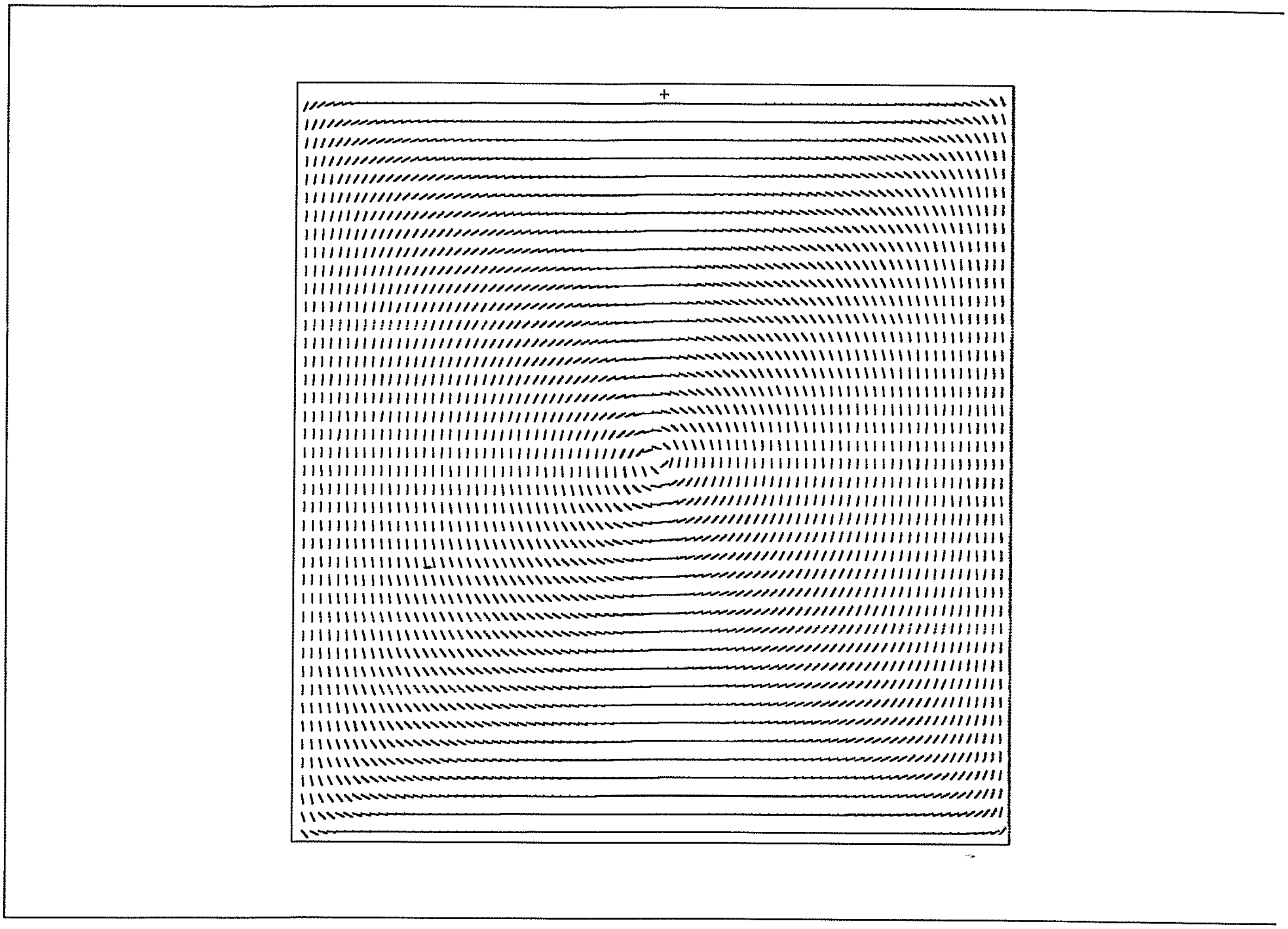



After 3325 iterations, computed J= 1371.57 ft Error= 0.000009

The term "Error" is computed as,
    Error = ABS|(Current J - Previous J ))/Current J |

For the last iteration computed, "Error" was computed as follows:
Error = ABS |( 1371.57 - 1371.55)/ 1371.57|=0.00000987

TORSIONAL PROPERTIES:



Torsional constant, J= 1371.57 $ft^4$
Maximum shear stress= 6.445 psf per T=1 lb*ft at X= 5.06 ft, Y= 9.86 ft